22nd Annual

AMC 8

(American Mathematics Contest 8)

Solutions Pamphlet

Tuesday, NOVEMBER 14, 2006

This Solutions Pamphlet gives at least one solution for each problem on this year's exam and shows that all the problems can be solved using material normally associated with the mathematics curriculum for students in eighth grade or below. These solutions are by no means the only ones possible, nor are they necessarily superior to others the reader may devise.

We hope that teachers will share these solutions with their students. However, the publication, reproduction, or communication of the problems or solutions of the AMC 8 during the period when students are eligible to participate seriously jeopardizes the integrity of the results. *Dissemination at any time via copier, telephone, e-mail, World Wide Web or media of any type is a violation of the competition rules.*

1. **(D)** Mindy's total was approximately $2 + 5 + 10 = \$17$.

2. **(C)** On the AMC 8 a student's score is the number of problems answered correctly. So Billy's score is 13. Because there is no penalty for guessing, if he wants to increase his score, he probably should fill in the last five answers.

3. **(A)** When Elisa started, she completed a lap in $\frac{25}{10} = 2.5$ minutes. Now she can complete a lap in $\frac{24}{12} = 2$ minutes. She has improved her lap time by $2.5 - 2 = 0.5$ or $\frac{1}{2}$ minute.

4. **(B)** Ignore the number of complete revolutions because they do not affect direction. One-fourth of the distance around the circle clockwise from west is north. Three-fourths of the distance counterclockwise around the circle from north is east. Chenille's spinner points east.

5. **(D)** Divide the larger square into 8 congruent triangles, as shown, 4 of which make up the smaller square.
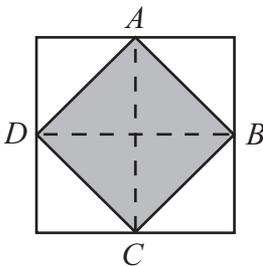
The area of the smaller square is $\frac{4}{8}$ or $\frac{1}{2}$ of the area of the larger square, so the area of the smaller square is equal to 30.

6. **(C)**
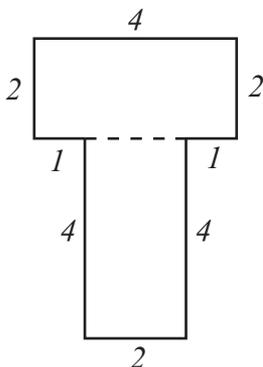
The perimeter is $4 + 2 + 1 + 4 + 2 + 4 + 1 + 2 = 20$ inches.

OR

Each rectangle has perimeter $= 2l + 2w = 2(4) + 2(2) = 8 + 4 = 12$ inches. When the two rectangles are positioned to form the T, a two-inch segment of each rectangle is inside the T and is not on the perimeter of the T. So the perimeter of the T is $2(12) - 2(2) = 24 - 4 = 20$ inches.

7. **(B)** Because circumference $C = 2\pi r$ and circle Y has circumference 8π, its radius is $\frac{8\pi}{2\pi} = 4$. Because area $A = \pi r^2$ and circle Z has area 9π, its radius is $\sqrt{9} = 3$. Ordering the radii gives $3 < \pi < 4$, so the circles in ascending order of radii length are Z, X and Y.

8. **(E)** Because $200 - 96 = 104$ of those surveyed were male, $104 - 26 = 78$ of those surveyed are male listeners.

	Listen	Don't Listen	Total
Male	78	26	104
Female	58	38	96
Total	136	64	200

The percentage of males surveyed who listen to KAMC is $\frac{78}{104} \times 100\% = 75\%$.

9. **(C)** Note that in each fraction, the numerator is the same as the denominator in the next fraction, so they divide. The product of $\dfrac{\cancel{3}}{2} \times \dfrac{\cancel{4}}{\cancel{3}} \times \dfrac{\cancel{5}}{\cancel{4}} \times \ldots \times \dfrac{2006}{\cancel{2005}} = \dfrac{2006}{2} = 1003$.

10. **(A)** When the area of a rectangle is 12 square units and the sides are integers, the factors of 12 are the possible lengths of the sides. In point form, the side lengths could be $(1, 12), (2, 6), (3, 4), (4, 3), (6, 2)$ and $(12, 1)$. Only graph A fits these points.

11. **(C)** The sum of the digits of a two-digit number is at most $9 + 9 = 18$. This means the only possible perfect square sums are $1, 4, 9$ and 16. Each square has the following two-digit possibilities:
$1 : 10$
$4 : 40, 31, 22, 13$
$9 : 90, 81, 72, 63, 54, 45, 36, 27, 18$
$16 : 97, 88, 79$
There are 17 two-digit numbers in all.

12. **(D)** Note that 70% of 10 is 7, 80% of 20 is 16 and 90% of 30 is 27. Antonette answers $7 + 16 + 27 = 50$ problems correctly out of 60 problems in all. Her overall score is $\frac{50}{60}$ or $83.\overline{3}\%$.

13. **(D)** Between 8:30 and 9:00 AM Cassie travels 6 miles. At 9:00 Cassie and Brian are only 56 miles apart. After 9:00, because they are both biking towards each other, the distance between them decreases at the rate of $12 + 16 = 28$ miles per hour. At that rate, it will take them $\frac{56}{28} = 2$ hours to meet. So they will meet at 11:00 AM.

14. **(B)** Bob takes $45 - 30 = 15$ more seconds per page than Chandra. So the difference in their total reading times is $760 \cdot 15 = 11{,}400$ seconds. Bob will spend 11,400 more seconds reading than Chandra.

15. **(C)** The ratio of time it takes Bob to read a page to the time it takes Chandra to read a page is 45:30 or 3:2, so Bob should read $\frac{2}{3}$ of the number of pages that Chandra reads. Divide the book into 5 parts, each with $\frac{760}{5} = 152$ pages. Chandra will read the first $3 \cdot 152 = 456$ pages, while Bob reads the last $2 \cdot 152 = 304$ pages.

<div align="center">OR</div>

If Chandra reads x pages, she will read for $30x$ seconds. Bob has to read $760 - x$ pages, and this takes him $45(760 - x)$ seconds. Because Chandra and Bob read the same amount of time, $30x = 45(760 - x)$.
Solving for x,

$$
\begin{aligned}
30x &= 45 \cdot 760 - 45x, \\
75x &= 45 \cdot 760, \\
x &= \tfrac{45 \cdot 760}{75} = 456.
\end{aligned}
$$

So Chandra will read the first 456 pages.

16. **(E)** The least common multiple of 20, 45 and 30 is $2^2 \cdot 3^2 \cdot 5 = 180$. Using the LCM, in 180 seconds Alice reads $\frac{180}{20} = 9$ pages, Chandra reads $\frac{180}{30} = 6$ pages and Bob reads $\frac{180}{45} = 4$ pages. Together they read a total of 19 pages in 180 seconds. The total number of seconds each reads is $\frac{760}{19} \cdot 180 = 7200$.

17. **(B)** Because the sum of a number from spinner Q and a number from spinner R is always odd, the sum of the numbers on the three spinners will be odd exactly when the number from spinner P is even. Because 2 is the only even number on spinner P, the probability of getting an odd sum is $\frac{1}{3}$.

18. **(D)** Four black and five white squares are visible on each of the six faces of the cube. So $\frac{5}{9}$ of the surface will be white.
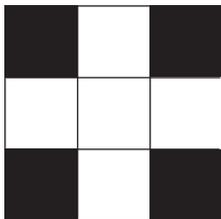
19. **(D)** Because triangles ABD and ECD are congruent and triangle ABC is isosceles, $EC = AB = BC = 11$. That means $BD = \frac{11}{2}$ or 5.5.

20. **(C)** Each of the six players played 5 games, and each game involved two players. So there were $\frac{6 \cdot 5}{2} = 15$ games. Helen, Ines, Janet, Kendra and Lara won a total of $4 + 3 + 2 + 2 + 2 = 13$ games, so Monica won $15 - 13 = 2$ games.

21. **(A)** Using the volume formula $lwh = V$, the volume of water in the aquarium is $100 \times 40 \times 37 = 148,000\,\text{cm}^3$. When the rock is put in, the water and the rock will occupy a box-shaped region with volume $148,000 + 1000 = 149,000\,\text{cm}^3$. The volume of the water and the rock is $100 \times 40 \times h$, where h is the new height of the water. The new volume $= 4000h = 149,000\,\text{cm}^3$, so the new height is

$$h = \frac{149000}{4000} = 37.25\,\text{cm}.$$

After adding the rock, the water rises $37.25 - 37 = 0.25\,\text{cm}$.

<div align="center">OR</div>

Because the shape of the rock is irrelevant, we may assume that the rock is shaped like a rectangular box with base measuring $100\,\text{cm} \times 40\,\text{cm}$ and height h cm. Using the volume formula, $100 \times 40 \times h = 1000$, so $h = \frac{1000}{100 \times 40} = 0.25\,\text{cm}$. When the rock is put into the aquarium, the water level will rise by $0.25\,\text{cm}$.

22. **(D)** If the lower cells contain A, B and C, then the second row will contain $A + B$ and $B + C$, and the top cell will contain $A + 2B + C$. To obtain the smallest sum, place 1 in the center cell and 2 and 3 in the outer ones. The top number will be 7. For the largest sum, place 9 in the center cell and 7 and 8 in the outer ones. This top number will be 33. The difference is $33 - 7 = 26$.

23. **(A)** The counting numbers that leave a remainder of 4 when divided by 6 are $4, 10, 16, 22, 28, 34, \ldots$. The counting numbers that leave a remainder of 3 when divided by 5 are $3, 8, 13, 18, 23, 28, 33, \ldots$. So 28 is the smallest possible number of coins that meets both conditions. Because $4 \times 7 = 28$, there are no coins left when they are divided among seven people.

<div align="center">OR</div>

If there were two more coins in the box, the number of coins would be divisible by both 6 and 5. The smallest number that is divisible by 6 and 5 is 30, so the smallest possible number of coins in the box is 28.

24. **(A)** We can decompose $CDCD$ into $CD \times 100 + CD = CD(101)$. That means that $A = 1$ and $B = 0$. The sum is $1 + 0 = 1$.

25. **(B)** There are one odd and two even numbers showing. Because all primes other than 2 are odd and the sum of an even number and an odd number is odd, the common sum must be odd. That means 2 must be opposite 59 and the common sum is $2 + 59 = 61$. The other two hidden numbers are $61 - 44 = 17$ and $61 - 38 = 23$. The average of 2, 17 and 23 is $\frac{2 + 17 + 23}{3} = \frac{42}{3} = 14$.